Paper 1: Mechanics and Properties of Matter

Module-3

Programme:	B.Sc.
Subject:	Physics
Semester:	I semester
University:	Davangere
Session:	02

Rotation of a Rigid body

Session 2-Syllabus

1. kinetic energy of a rotating body

2.Relation between angular momentum & moment of inertia (L = I ω)

3.Conservation of angular momentum

$\left[\tau = \frac{dL}{dt} = 0\right]$ **and illustrations.**

Learning Objectives

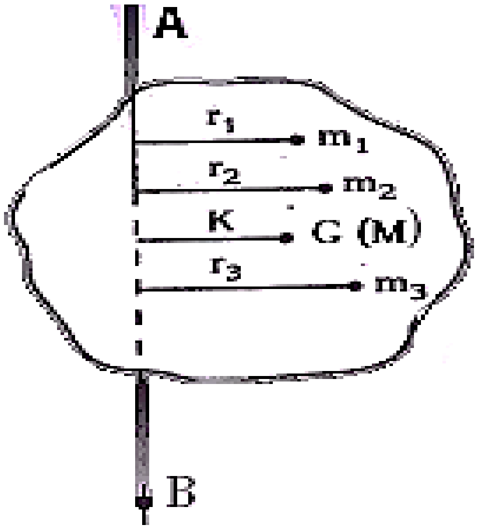
- After this session, students should be able to:

- Understand the fundamental concepts of a Rigid Body,
- Understand basic properties of kinetic energy of rotating body.
- Understand Angular momentum
- Understand the concept Relation between angular momentum & moment of inertia
- Understand the concept of torque

•Expression for kinetic energy of rotation:

Consider a rigid body rotating about a given axis ‘A’.It is madeup of *n* number of particles of m_1, m_2, m_3, etc at distance r_1, r_2, r_3, etc from the axis ‘AB’.

If the body rotates with an angular velocity ω then, the different particles complete their circular path with different radii in the same time. Thus, the linear velocities (*v*) of particles are different.



If v_1, v_2, v_3, etc are the linear velocity of the particle then the total kinetic energy of the different particles is given by,

$$KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \ldots + \frac{1}{2} m_i v_i^2$$

But $v = r\,\omega$

$$v = r_1\,\omega \qquad v_2 = r_2\,\omega \qquad v_i = r_i\,\omega$$

Then we can write,

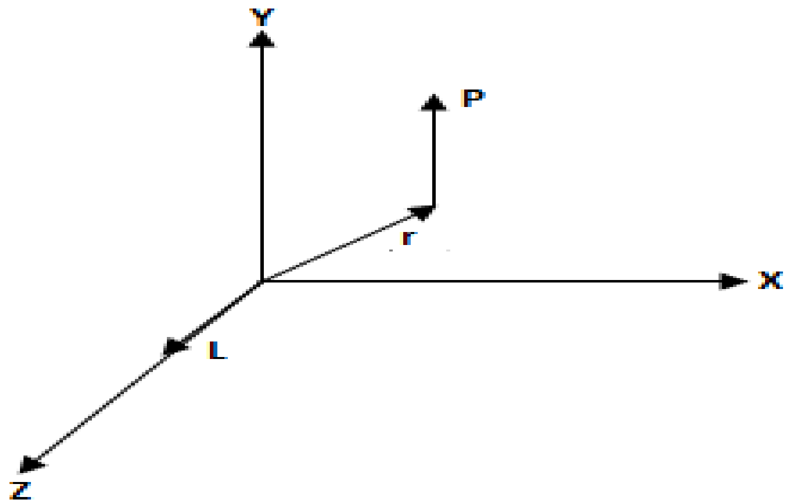
$$KE = \frac{1}{2} m_1 r_1^2\omega^2 + \frac{1}{2} m_2 r_2^2\omega^2 + \ldots\ldots + \frac{1}{2} m_i r_i^2\omega^2$$

$$KE = \frac{1}{2}\omega^2 \left[m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_i r_i^2 \right]$$

$$KE = \frac{1}{2}\omega^2 \left[\sum m_i r_i^2 \right] = \frac{1}{2}\omega^2 I = \frac{1}{2} I \omega^2$$

$$KE_{rot} = \frac{1}{2} I \omega^2$$

Angular Momentum



Consider a particle of mass m located at the vector position $\vec{r}$ and moving with linear velocity $\vec{v}$. Then the angular momentum of the particle is the product of position vector $\vec{r}$ and linear momentum $\vec{p}$. That is $\vec{L} = \vec{r} \times \vec{p}$

Since $p = mv$, the magnitude of the angular momentum is given by $\boldsymbol{L = mvrsin\theta = mr^2\omega sin\theta}$**.** Where θ is the angle between r and p. If θ= 0, then L=0, implies that if particle moving along the line passing through the origin, then particle will not possess angular momentum. On the other hand if $\theta = 90^0$, then $\boldsymbol{L = mr^2\omega}$,

(L = Iω Where I = mr^2 & v = rω)

That is the particle will rotate about the origin in XY plane.

Thus "the angular momentum of the body consisting of n number of particles can be defined as the vector sum of angular momentum of individual particles.

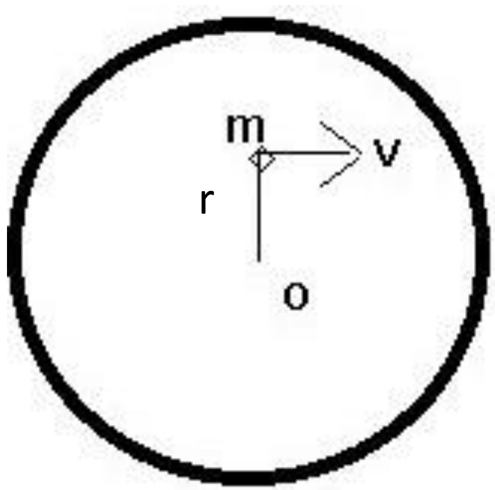
$$\boldsymbol{ie\,\vec{L} = \sum_{i=1}^{i=n} r_i \times p_i}$$

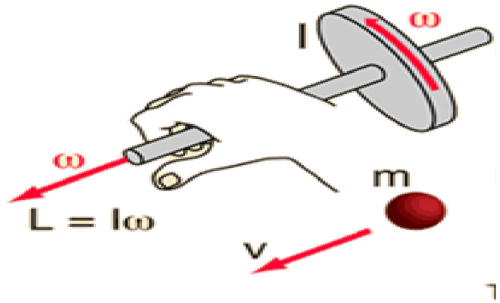
Relation between angular momentum & moment of inertia (L = I ω).

Moment of Inertia: When a rigid body rotates about an axis it has tendency to oppose change of uniform rotation about it axis. This tendency of the body is called rotational inertia or moment of inertia of the body about its axes of rotation.

Moment of Inertia of a particle : Momentum of inertia (M.I) of a particle of mass ‘m’ about an given axis of rotation is defined as product of mass and the square of distance ‘r’ the particle from the axis of rotation

$\mathbf{I = m\, r^2}$ Kgm^2





Angular Momentum: The angular momentum of a body about an axis is defined as the product of its linear momentum and the perpendicular distance from the axis of rotation to its line of motion .

Angular Momentum of a body $L = mvr$

Here r = Distance of the line of motion from the axis.

Linear momentum = P= mv

Angular momentum of a particle is the moment of linear momentum about that point.

$$L = mvr = mr\omega r = mr^2\omega$$

$\mathbf{L = I\omega}$ Where $I = m\,r^2$ & $v = r\omega$

Here we have to note that, like Linear momentum of a body,

The angular momentum is the product of Moment of inertia of the body and

its angular velocity.

Torque: The torque is the rate of change of angular momentum.

Torque is the measure of the force that can cause an object to rotate about an axis. Force is what causes an object to accelerate in linear kinematics. Similarly, **torque** is what causes an angular acceleration. Hence, **torque** can be defined as the rotational equivalent of linear force.

$\boldsymbol{\tau} = \frac{dL}{dt} = \frac{d(I\omega)}{dt} = I \frac{d\omega}{dt} = I\alpha$ **Torque =** $\boldsymbol{\tau} = I\alpha$ for rotational motion.

This is similar to force being equal to product of mass and acceleration.

The Law of conservation of angular momentum:

The equation $\tau = \frac{dL}{dt}$ states that the external unbalanced torque is equal to the rate of change of angular momentum.

If the external torque is zero,

$$\tau = \frac{dL}{dt} = 0$$

The angular momentum L is a constant.

$$L = I\omega = \text{Constant}$$

Definition of "The Law of conservation of angular momentum":

If there is no external torque acting on a system ($\tau = 0$), The angular momentum of a system remains constant.

Where I is the moment of inertia of the body about the axis, ω is its angular velocity.

That is $I\omega = a\ constant$ or $I \propto \frac{1}{\omega}$.

$L = I\omega$ = Constant or $I_1\omega_1 = I_2\omega_2$ = Constant

Illustrations:

1. **Motion of a Planet around the Sun:** We know that planets move in elliptical orbits around the sun, with sun at one of the foci. The gravitational force on the planet is directed towards the Centre of the sun. So the force is central force. Due to this fact, the angular momentum of the planet will be constant.

1. **Diving, Skating, Ballet dancing:** In all these cases the performer uses the principle of conservation of angular momentum. The angular momentum of a body is given by

$$L = r \times p = r \times mv = r \times mr\omega = mr^2\omega = I\omega$$

Multiple Choice Questions

A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of a smooth incline and released. Least time will be taken in reaching the bottom

(a) The solid sphere
(b) The hollow sphere
(c) The disc
(d) All will take same time

Multiple Choice Questions

Which of the following has the smallest moment of inertia about the central axis if all have equal masses and radii?

(a) Ring

(b) Disc

(c) Spherical shell

(d) Sphere

Answer:(d)

Multiple Choice Questions

The dimensions of moment of inertia are:

(a) $[M^1 L^2 T^1]$

(b) $[M^0 L^2 T^0]$

(c) $[M^0 L^2 T^2]$

(d) $[M^1 L^2 T^0]$

Answer:(d)

Multiple Choice Questions

A couple produces:

(a) No motion

(b) Linear and rotational motion

(c) Purely rotational motion

(d) Purely linear motion

Answer:(c)

Multiple Choice Questions

Analogue of mass in rotational motion is:

(a) Moment of inertia

(b) Angular momentum

(c) Gyration

(d) None of these.

Answer:(a)

Work Sheet

- **Two mark questions:**

1. Derive the expression for kinetic energy of rotation.
2. Distinguish angular momentum & moment of inertia
3. Define the Law of conservation of angular momentum

Reference Books:

i) Elements of Properties of Matter – By D.S.Mathur

ii) Physics for Degree Students - B.Sc First Year
C.L.Arora , P.S. Hemne

iii) College Physics-Vol-1 By N.s Sundar Rajan and others